Performance Analysis of Fractional Fourier Transform for MIMO-OFDM Systems

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) has become a popular technique for transmission of signals over wireless channels and has been adopted in several wireless communication standards. Multiple-input multiple-output (MIMO) antennas can be combined with OFDM to achieve diversity gain and/or to increase system spectral efficiency. MIMO technology uses multiple antennas at both the transmitter and receiver to increase the system's capacity and improve its performance by reducing interference and improving signal quality. Additionally, OFDM modulation divides the data stream into multiple subcarriers and modulates them separately. This helps to mitigate the effects of channel fading and improves the system's spectral efficiency. MIMO OFDM systems combine these two technologies to provide high-speed, reliable wireless communication over a wide range of distances. In this work, we investigate the performance of Fractional Fourier Transform (FrFT) as a suitable replacement of the Fast Fourier Transform (FFT) used in conventional OFDM. The bit error rate (BER) performance of conventional OFDM is first investigated, using BPSK, 16PSK and 16QAM, to determine the best performing modulation scheme. A FrFT based MIMO-OFDM system is then observed and compared to FFT based MIMO-OFDM system under 16-QAM modulation over AWGN. The simulation results indicate that BER performance of FrFT based MIMO-OFDM system is better than the FFT based MIMO OFDM system.**

Keywords— Wireless Communication, MIMO-OFDM, FFT, FrFT, AWGN, BER, SNR

I. Introduction

The basic principle of a MIMO-OFDM wireless communication system is to divide the available frequency band into multiple subcarriers and use OFDM modulation to transmit data over these subcarriers. Each of the subcarriers carries a small portion of the data which is distributed across all the subcarriers. This is done in a way that maximizes the spectral efficiency of the system. Multiple antennas are used at both the transmitter and receiver end to exploit the spatial diversity of the wireless channel. The signals transmitted from each antenna are then combined at the receiver, which improves the quality of the received signal and increases the data rate. The MIMO-OFDM system uses advanced signal processing techniques to mitigate the effects of multipath fading and interference [1]. These techniques improve the signal quality, reduce the error rate, and increase the overall performance of the wireless communication system. The purpose of the MIMO-OFDM wireless communication system is to provide a high-speed and reliable wireless communication link between two or more devices. The system is able to achieve the following:

* Increased data rate: A MIMO-OFDM system can provide high data rates by using multiple antennas at both the transmitter and receiver ends. The system exploits the spatial diversity of the wireless channel to transmit and receive multiple streams of data simultaneously.

* Improved spectral efficiency: Dividing the available frequency band into multiple subcarriers allows the system to transmit data over multiple channels simultaneously. This maximizes the use of the available spectrum and improves the spectral efficiency of the system.

* Enhanced quality of the wireless link: The MIMO OFDM system uses advanced signal processing techniques to mitigate the effects of multipath fading and interference. These techniques improve the signal quality, reduce the error rate, and increase the overall performance of the communication system.

Conventional OFDM is a Fast Fourier Transform (FFT) based system. It uses IFFT (Inverse FFT) blocks in the transmitter for modulation and FFT blocks in the receiver for the demodulation [2]. Fractional Fourier Transform (FrFT) is a time frequency distribution and an extension of the classical FFT, used in OFDM. It has been suggested in previous works [3] [4], that FrFT offers improvements in implementations in different fields in place of FFT. On comparing the received image quality of hybrid combination of massive MIMO and OFDM augmented with FFT, FrFT and DWT transforms, it is observed that the received image quality in the case of FrFT augmented OFDM is much better than the OFDM system augmented with FFT [4]. Additionally, the FrFT has been adapted to a variety of other fields, such as digital watermarking, image compression, and more [3]. Using MIMO technology with OFDM is an exciting solution for next-generation broadband wireless communication systems [5].

II. ANALYSIS

A. Multiple Input Multiple Output (MIMO)

Multiple antennas can be used at the transmitter and receiver, an arrangement called a multiple-input multiple-output (MIMO) system. A MIMO system takes advantage of the spatial diversity that is obtained by spatially separated antennas in a dense multipath scattering environment. MIMO systems may be implemented in a number of different ways to obtain either a diversity gain to combat signal fading or to obtain capacity gain. Increasing the number of antennas in the system not only enhances the sum rate spectral efficiency but also simplifies the required signal processing at both the transmitter and receiver [4].

There are three main components of a MIMO system, Transmitter, Receiver and the Channel. A simple block diagram of MIMO system is depicted in Figure 1. Nt represents the multiple antennas at the transmitter side and Nr symbolizes the multiple antennas at the receiver side. Hence, the MIMO wireless communication channel is constituted by (Nr x Nt) propagation links.

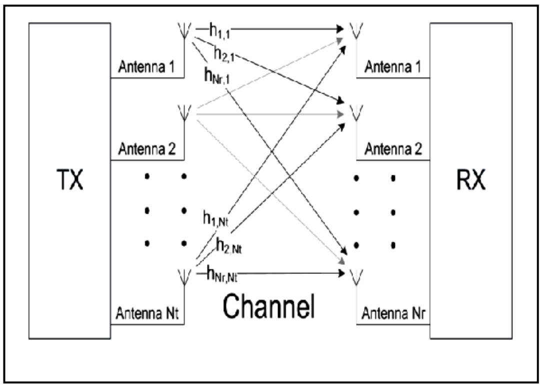

Figure 1: MIMO system block diagram

MIMO systems use multiple signal paths called spatial streams to transmit signals simultaneously. This is made possible through multipath propagation, where transmitted signals take different paths due to reflections, scattering, and diffraction in the environment. MIMO takes advantage of these independent signal paths to achieve two major benefits:

* Spatial Diversity: MIMO improves the reliability of wireless links by reducing the impact of fading and interference. Each antenna provides an independent observation of the transmitted signal, allowing for redundancy in case one path experiences degradation.

* Spatial Multiplexing: By transmitting different data streams from multiple antennas simultaneously, MIMO realizes increased data throughput. Each stream occupies the same frequency band but is separated spatially, allowing multiple signals to be received independently at the receiver.

B. Othorgonal Frequency Division Multiplexing (OFDM)

Long-range communication systems require efficient methods for data transmission that can handle multipath fading, interference, and bandwidth limitations. Orthogonal Frequency Division Multiplexing (OFDM) is a method of data transmission where a single information stream is split among several closely spaced narrowband subchannel frequencies instead of a single wideband channel frequency. The advantages of OFDM are its ability to cope with multipath fading as well as provision for bandwidth efficiency.

The main concept of OFDM is the Orthogonality of subcarriers. OFDM transmits symbols that have long time duration, which is less or equal to the maximum delay spread. To eliminate Inter symbol interference (ISI), guard intervals are used, by adding Cyclic Prefix (CP) between OFDM symbols [2].

The mathematical representation of OFDM relies on the Discrete Fourier Transform (DFT) and its inverse (IDFT). The Inverse Fast Fourier Transform (IFFT) implements this operation, while Fast Fourier Transform (FFT) is used at the receiver for demodulation [6]. The IFFT converts the frequency-domain signal into a time-domain signal for transmission. FFT reverses the operation, recovering the transmitted data in the frequency domain.

In an OFDM system with N subcarriers, the individual data streams are first passed through OFDM modulators which perform the IFFT, and a cyclic prefix of length P is then prepended. Under this condition, a linear convolution of the transmitted sequence and the channel is converted to a circular convolution. As a result, the effects of the ISI are easily and completely eliminated. The resulting OFDM symbols of length $N + P$ are launched simultaneously from the transmit antennas.

In the receiver the individual signals are passed through OFDM demodulators which first discard the CP and then perform an N-point FFT. The outputs of the OFDM demodulators are finally separated and decoded.

AWGN Channel

High data rate communication over additive white Gaussian noise channel (AWGN) is limited by noise [7]. The received signal in the interval 0? t? T may be expressed as

r(t)=sm(t) + n(t)

Where n(t) denotes the sample function of additive white Gaussian noise (AWGN) process with power-spectral density.

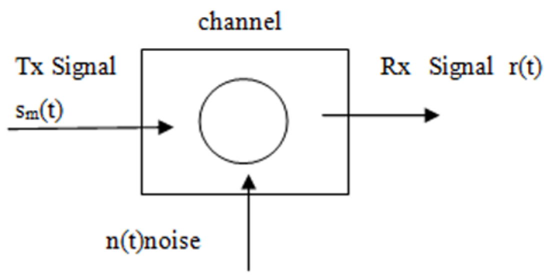

Figure 2: Model for received signal passed through AWGN

AWGN channel model is widely used in studying OFDM. In this model there is only linear addition of white noise with a constant spectral density and Gaussian distribution of the amplitude [7].

Cyclic Prefix

Cyclic prefixing refers to the prefixing of a symbol, with a repetition of the end. OFDM makes use of cyclic prefixes to combat multipath by making channel estimation less complex. The cyclic prefix is created so that each OFDM symbol is preceded by a copy of the end part of that same symbol. The addition of the cyclic prefix adds robustness to the OFDM signal. The guard interval introduced by the addition of cyclic prefix to the system enables the reduction of the effects of inter-symbol interference.

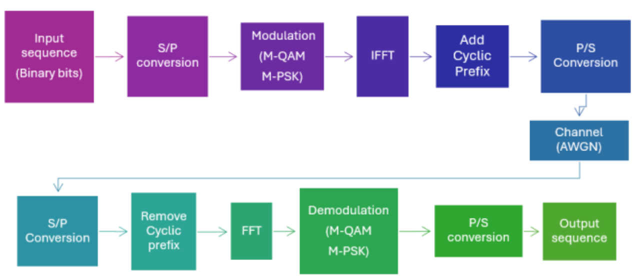

Figure 3: Simple Diagram of OFDM system

Figure 3 shows a simplified block diagram of an OFDM system with the operations at the transmitter and receiver sides. At the transmitter, a binary input sequence undergoes serial to parallel conversion and is then modulated to QAM symbols with symbol interval, Ts. The serial to parallel converter collects the serial data symbols *S(n)* into length *N* vector *S= [S (0) S (1)   S (N-1)]T* with *N* being the number of subcarriers. This vector is then modulated by an inverse fast Fourier transform (IFFT) into an OFDM symbol sequence as follows:

$$s(n) = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} S(k)e^{j\frac{2\pi nk}{N}}, \quad 0 \le n \le N-1$$

At the receiver, the cyclic prefix is removed from the received sequence and the OFDM demodulator performs an N-point FFT on the sequence to yield the demodulated output sequence.

III. SYSTEM MODEL

MIMO-OFDM technique has proven to be effective for combating multi-path delay spread and increasing system capacity. Conventional OFDM systems offer robust performance, but their effectiveness can be enhanced through advanced signal processing techniques.

FrFT based OFDM system

Fractional Fourier Transform (FrFT) is a time frequency distribution and an extension of the classical Fast Fourier transform (FFT), used in conventional OFDM. The existence of the FrFT has been widely studied and it is found that the FrFT of a signal x(t) exists under the same conditions in which its Fourier transform exists [8].

The FrFT is the generalized formula for the Fourier transform that transforms a function into an intermediate domain between time and frequency. We may interpret it as a rotation operator in the time-frequency plane. This property makes the FrFT especially suitable for the processing of linear frequency modulation (LFM) also known as chirp like signals or the signals passing through the linear time-varying system [9]. The fractional Fourier transform of order *a* of an arbitrary function x(t), with and angle α is given as:

$$X_\alpha(u) = \int_{-N/2}^{N/2} x(t)K_\alpha(t, u)dt$$

where $K_\alpha(t, u)$ is the transformation Kernel and $\alpha = a\pi/2$

The inverse FrFT (IFrFT) can be expressed as:

$$x(t) = \int_{-N/2}^{N/2} X\alpha(u)K_{-\alpha}(t, u)du$$

The signal x(t) in above can be interpreted as decomposition to a basis formed by the orthonormal LFM functions in the u domain. The u domain is usually called the fractional Fourier domain, while the time and frequency domain can be considered as its special cases when a = 0 and a = 1 respectively.

The FrFT based OFDM system is similar to the conventional OFDM system with an FrFT modulator replacing the FFT modulator. Because MIMO technology is able to bring together many smaller antennas to function as one, it can receive and send multiple OFDM sub-signals in a way that significantly increases the bandwidth needs for each user as needed.

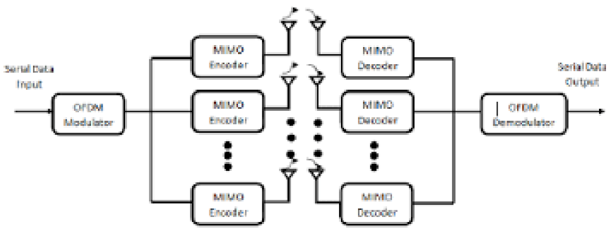

Figure 4:Block diagram of MIMO-OFDM system

A simple block diagram of MIMO-OFDM system is shown in Figure 4, where at the OFDM modulator, in the transmitter, instead of using IFFT, IFrFT is used. At the receiver, FrFT is used at the OFDM demodulator. The development of OFDM in mix with MIMO offers an attractive interface and gives a suitable solution to the current demands in wireless communication systems.

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IV. IMPLEMENTATION AND RESULTS

In this work MATLAB R2022a is used as the simulation environment. MATLAB is a strong mathematical tool that provides help to engineers to solve, model, simulate the problems, and find solutions to mathematical equations. It is a standard engineering tool that performs many different tasks using different toolboxes relevant to different particular cases e.g. Control systems, signal processing, image processing, and communication systems. In research and universities, it provides platform for learning and comparison of theatrical hypothesis and simulated values [10].

We simulate the FrFT MIMO-OFDM system with the parameters given in Table 1.

Table 1: Simulation Parameters

Parameter	Value
OFDM subcarriers	128
Number of symbols	1000
Cyclic prefix	16
Modulation schemes	BPSK, 16-PSK, 16-QAM
Range of SNR (dB)	0-30
MIMO channel	2*2
Alpha for FrFT	0.9

First, conventional OFDM that uses IFFT and FFT is simulated over AWGN, to see which among BPSK, 16-PSK and 16-QAM gives better performance in terms of BER vs SNR. The results as shown in Figure 5 indicate that 16-QAM performs the best among the three modulation schemes. A similar simulation is then conducted, using FrFT based OFDM with the same modulation schemes. 16-QAM again performs better than the other two schemes as shown in Figure 6

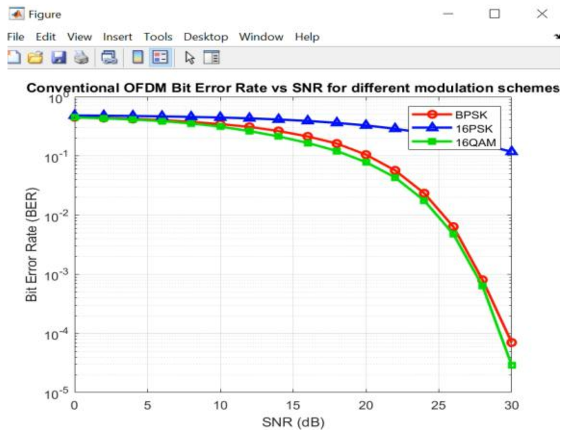

Figure 5Comparison of BER performance of different modulation schemes for conventional OFDM

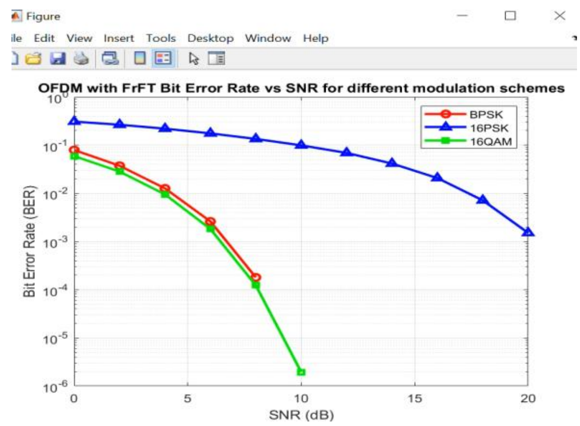

Figure 6Comparison of BER performance of different modulation schemes for FrFT based OFDM

From the above findings, 16-QAM is therefore picked to be used for simulation of the MIMO OFDM system that uses Fractional Fourier Transform. The result of this simulation is shown in Figure 7. It is evident that using FrFT gives better performance with regard to bit error rate, when used in a MIMO OFDM system, compared to the use of FFT.

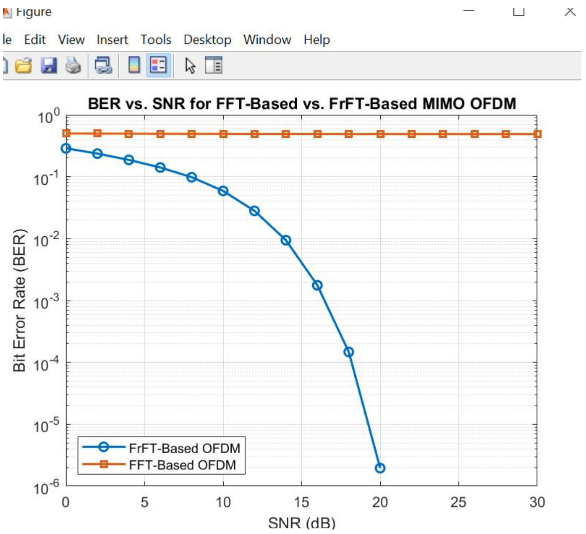

Figure 7Figure 7: Comparison of BER performance of FrFT based MIMO OFDM and FFT based MIMO OFDM

V. CONCLUSION

This work explores the theory and workings of a hybrid system that puts together OFDM and MIMO communication technologies, under different transforms, i.e FFT and FrFT. The analysis and simulation is presented in a channel with AWGN for simplicity and 16-QAM as the preferred modulation scheme. It is very evident from the simulation results that the MIMO OFDM system that employs the use of FrFT outperforms the one that uses FFT.

Future works on this may aim to investigate the results in different and more complex channel conditions such as underwater acoustic channels, and doppler scenarios that simulate a moving transmitter and receiver. The use of different number of antennas for extended MIMO technologies to NxN MIMO can also be pursued.

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